Systematic Risk Outliers and Beta Reliability in Emerging Economies: Estimation-Risk Reduction with AZAM Regression

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ABSTRACT

Beta is traditional market risk measure used in finance. There are verities of Beta estimation methods and despite multi dimensional criticism and reservations regression is still widely used method. Based on the idea of ranking regressions with multi criteria decision making tool named fuzzy-AHP intelligent zax, put forth in a recent article published in the lecture notes of software engineering, this article extends the idea in order to get aggregated zoom AHP-mated (AZAM) regression for beta estimation. AZAM based Beta results are derived from regular beta regressions from a published article for 17 countries. Finally the article has compared the results for AZAM beta with regular regression beta.

Keywords: Market Beta, Investment Risk, Data Outliers, Estimation Risk, Multi criteria decision making,

INTRODUCTION

Predicting stock returns from historical data has a long history as it has importantfinancial consequences. An investor who perfectly predicted the monthly return or sign of the Standard & Poor's 500 Index (S&P500) from January 1989 to March 2007 and implemented a simple monthly long-short switching strategy with a cost level of 0% (1%) [2%] per switch would have earned 40.9% (35.0%) [29.1%] p.a., compared to 11.8% p.a. of a passive S&P500 investor (Steiner; 2009). Market risks directly affect returns thus predicting market risk beta has similar financial consequences. This is evident from Scherer (2010) which, using 72 data points from years 2000 to 2007 of nine publicly traded asset management firms, estimated revenue beta using various panel regression models (i.e. un-pooled data regressions, classical pooling (stacking), fixed effects, random effects and random coefficients models) and showed that asset management revenues carry substantial market risks, besides traditionally thought of operational risks. Market beta directly affects asset manager fees by their impact on returns. It affects both asset- based and performance-based fees, and they correspond roughly to beta (general economic and market exposure) and alpha (out performance vs. a risk-adjusted benchmark) risks. Thus market or beta risk exposures can be very costly for asset management companies, as noted by Doherty (2000) as "quoted from Scherer" (QFS hereafter) and thus hedging these risks could be highly beneficial.

Beta is widely used quantity in investment analysis and it is used to apportion risk to the market (Tofallis; 2008). But what exactly beta or market risk is? In statistics Roman letters are referred for measured or estimated values based on a sample of data, whereas Greek symbols are referred for the true, but unknown population values. In finance, the term 'beta' refers to the slope in a linear relationship fitted to data on the rate of return on an investment and the rate of return of the market (usually the market index). The relationship can be stated in one of the two forms: (1) $R_i = \alpha + \beta R_m$ and (2) $R_i - r_f = \alpha + \beta (R_m - r_f)$. R_m here is the rate of return on the market or an index of the market (as proxy), R_i represents the rate of return on an investment, r_f is risk-free investment (such as lending to the government) and $(R_m - r_f)$ is called excess returns (or risk premium) which

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is the rate of return above and beyond r_f. First (second) equation can be a line fitted to the data (characteristic line for that investment), with α and β being the intercept and slope of that line. As the linear relationship with the market returns in equation (1) will not be perfect i.e. most points will not lie on the line, so an error term (ε) is required when referring to particular data points. The final form is presented as (3) $R_i = \alpha + \beta R_m + \varepsilon$ and is refer to as a standard Capital Asset Pricing Model (CAPM). In the CAPM equation, βR_m is return explained by market variations (or explained sum of squares or ESS in ordinary least square (OLS) terms) and ε explains non-market variations (or Residual sum or squares or RSS). The 'market' in CAPM refers to the universe of all investments, which includes foreign equities, bonds, land, property, gold, derivatives, foreign currencies etc. and usually is represented by a proxy of index (DJIA or S&P) (Tofallis; 2008).

OLS continues to be by far the most frequently used method even when it is obviously inappropriate. As a result, hundreds if not thousands of regression lines with too-small slopes are being published annually (Riggs et el.; 1978). OLS results are, without doubt, severly impaired in the presence of outliers. Various financial shocks in general and subprime crisis of 2008 in particular, which created an environment of systematic and contagious crisis, have sent waves of market instability to the economies beyond the boundaries of its origin can be blamed to have created outliers in the financial data. Estimation of the market risk or beta by using OLS with such a dataset is bound to produce estimation risk. This estimation risk should be reduced, if not eliminated, in order to have better prediction of future returns of portfolios using resulted beta from such regression. There are various methods available in literature to handle the estimation risk for data with outliers and different methods are good only in a particular situation. Nathaphan (2010) has estimated three different regressions for various countries on three different datasets: total period (TP) data from 1995 to 2008, sample period 1 (SP1) data from 1995-2001 and sample period 2 (SP2) data from 2002-2008. The study has used a Beysian fix for estimation risk in portfolio performance. Our study has focused on the issue of reduction in estimation risk by getting an aggregated zoom AHP-mated (AZAM) regressions based on the ideas of getting ranks based on fuzzy weights (called FAIZ approach) (Anjum; 2014b), step wise utility function (SWUF) based scores of Kramer (2008), Be-ALAM regression concept from Anjum (2014d), ranking of various alternatives from Anjum (2003a&b, 2014a&c) along-with data inputs from Nathaphan (2010). The article is organized into five sections, including this section of introduction. Second section reviews literature about theory of beta, estimation techniques, various interpretations of beta and its uses (especially in the predictability of returns). Third section reviews various criticism extended to either measures of beta or OLS estimates. Fourth section named focusing data outliers describes the effects of outliers on estimation results and various ways to handle outliers. Section five focuses on description of methodology introduced in this article to handle the issue of data outliers in beta estimation, description of data inputs used to calculate results from the proposed methodology and the tables for the calculated results from AZAM and AZEM regressions. Finally, in the section six, results have been discussed and also conclusion is provided.

REVIEW OF LITERATURE

The theory of beta (from Sharpe's 1963 paper) as based on the theory of firm with debt and equity (assuming absence of taxes) as financing options giving a balance sheet identity (Riedl et el.; 2009). Firm's balance sheet identity reads as total assets (A) equals equity (E) plus total debt (D). A macro level consolidated balance sheet (CBS) identity based on contingent claims or various entities e.g. central bank, Fed and banking system has been presented in Chaudhary and Anjum (1996). This article will discuss firm's accounting identity. Scaling this firm's accounting identity by A, the weighted average beta for the firm is $\beta A\ (A/A) = \beta E\ (E/A) + \beta D\ (D/A)$ and solving this for the equity beta leads to equation (4): $\beta E\ (E/A) = \beta A\ FVA - \beta D$ Leverage; where

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FVA is the firm's fair value of assets, βE (E/A) is adjusted beta and Leverage is the firm's debt divided by total assets. Variance of estimated returns obtained from CAPM equation (with α and ε) can be stated as $Var(Ri) = Var(\alpha) + Var(\beta R_m) + Var(\varepsilon)$. Assuming constant α & β, we get equation (5): $Var(Ri) = Var(\beta R_m) + Var(\varepsilon)$ = market risk (or systematic or diversifiable risk)+ investment specific risk (or Unsystematic or non-diversifiable risk). For very well diversified portfolios, nonsystematic risk tends to go to zero and the only relevant risk is systematic risk measured by beta (Elton et al (2003) as 'quoted from Tofallis (2008) (QFT hereafter). If the market risk is diversifiable (that is, fully idiosyncratic to the individual firm), then the allocation of assets across different categories should have no relation to equity beta. Finance theory, however, suggests that information risk (i.e. the uncertainty regarding valuation parameters for an underlying asset) is reflected in firms' equity betas and studies like Clarkson and Thompson (1990) and Easley and O'Hara 2004) as "quoted from Reidl" (QFR hereafter) suggest that information risk is not diversifiable. And in such an economy, uncertainty surrounding the payoff distribution of a portfolio and equity beta can have a positive relation. Banz (1981) and Reinganum and Smith (1983) (QFR) indicated that information risk should be diversifiable in an economy. In general, if parameter uncertainty about expected future cash flows is uncorrelated across a sample of low information assets, portfolio formation can increase precision and eliminate any effects on systematic risk. A measure of information risk relevant to financial institutions based on reporting requirements of U.S. accounting standard setters adopted in Statement of Financial Accounting Standards (SFAS) 157 (Fair Value Measurement), which assess the financial instruments across three levels (level 1, 2, and 3) as defined in Reidle et el. (2009). These level 1, 2, and 3 portfolios are assumed to have securities that have equal co-movement (on average) with the market. The study has hypothised, based on Lambert, Leuz, and Verrecchia (2007) evidence that a firm's beta from the CAPM is a function of its information quality, that if required fair value designations appropriately capture this risk, then we expect that the implied beta is increasing across asset portfolios designated as level 1, 2, and 3. The study has also used adjusted beta decompositions (i.e. [ρ_{im} (E/A)] and [σ_i / σ_m (E/A)] as alternative dependent variables to better isolate the association between risk and the fair value level 1, 2, and 3 designations. The paper has negated the evidence that bank asset structure leads to greater opaqueness, (a justification for regulatory intervention) and proved that level 3 assets have higher systematic risk and lead to higher information asymmetry.

Shareholders do not want to be exposed to market beta by investing in asset management companies rather they want to participate in these companies' alpha generation (Steiner; 2009). Thus the main thing is the predictability of returns and literature tells us that it is possible. Majority of studies which explore the relationship between macroeconomic factors and equity prices, variable selection and empirical analyses is based on economic rationale, financial theory and investors' intuition and studies in the field of return predictability concentrates on predicting the market premium, apply a linear model and use one of the three approaches to predict premiums. Most popular approach tests the predictive power of one single variable e.g. Rozeff (1984), Giot (2005), Lettau and Ludvigson (2001) and Rapach and Wohar (2006) (refer to Steiner; 2009) (RTS hereafter)). The second approach applies multifactor. The third, least used, method is model selection approach. All these papers focus on an earlier sample period, few predictive models and apply a different methodology. However, many papers on the performance of investment funds, investment foundations, pension funds, and private investors show that almost no investment vehicles beat the market or have timing abilities thus raising questions about the value of the evidence for predictability. Besides, data-mining biases are most likely when testing hypotheses by digging down the same sample of data again and again (Lo and MacKinlay; 1990; RTS). These data mining biases can be evaluated using the framework utilising accuracy, robustness and predictability (see Anjum 2013a, 2014a) to rank the results gained each time after the same data visits as "alternatives". Steiner (2009) has introduced a new approach

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and investigates the out-of-sample predictability of the monthly market premium, by evaluating the aggregated results from the 1,024 models, by splitting the sample between an estimation period and an evaluation period, and has used the McCracken (2004) MSE-F-statistic (MSE-F) which uses difference between root mean squared error of the conditional model ($RMSE_c$) and of the unconditional model ($RMSE_u$) for comparing the forecast of two models. If RMSE>0 and MSE-F-statistic is significant, the predictions of the conditional model are significantly superior to the predictions of the unconditional model. As MSE-F has a nonstandard distribution, the study bases its inference on bootstrap procedure. Bootstrap is appropriate for relatively small samples, fairly high number of predictive variables, and overlapping observations. The study estimates all these equations via OLS and then computes model selection measures i.e. adjusted R^2, Akaike information criterion (AIC), and Schwarz information criterion (BIC), none of which has added any value compared to the unconditional model. All RMSEs are negative.

Besides these various approaches to predictability of returns, there are various estimation methods for beta from traditional models. The textbook way of estimating beta uses ordinary least squares (OLS) regression and the resulting slope or standard beta is (= $\rho * \sigma_i / \sigma_m$) or ($Cov_{im} / \sigma_i * \sigma_m$) where ρ (σ) [Cov] are correlation between (standard deviations of) [covariance between] the rate of returns and σ_i / σ_m is relative volatility. Some studies try, including Blume's beta (a weighted average of standard beta and one) and Vasicek's beta (a weighted average of standard beta and the average beta for a sample of stocks), to capture, as per literature suggestions, the tendency for standard beta values to approach the value of unity over time (QFT). We could alternatively estimate equation (4) including an intercept and either excluding or including other assets. Under this specification, the coefficients for assets measured at fair value capture the incremental beta relative to other assets. Reidl et. el. (2009) estimated a single-factor CAPM (e.g. of Sharpe (1964) or Black (1972)) to obtain banks' equity beta. Independent variables, in this study, includes bank's assets (decomposed into reported levels - 1 (2) [3] – and corresponding fair values FVA1 (FVA2) [FVA3] approximated by reported book values), all other assets (OA) and debt financing (Leverage) where all variables have been scaled by the firm's total assets. It also measured equity, debt, and other assets using book values and using identity A1 + A2 + A3 + OtherAssets = E + D. Based on equation (4) all betas (except debt financing beta) are supposed to be positive (negative).

CRITICISM OF BETA MEASURES AND OLS ESTIMATES

Various criticisms have been extended to traditional beta measures. Many investors do not hold well-diversified portfolios, and so for them market risk is an incomplete risk measure (Camp and Eubank (C&E); 1985) (QFT). An analysis with the 30 stocks making up the Dow Jones Industrial Average (an index or a weighted average of its components) where half of them had standard betas less than unity and index is supposed to be less variable than its components (central limit theorem). But evidence is contrary to this fact. So C&E suggested the use of ratio of standard deviations (called 'beta quotient') as a measure of risk and return performance of a portfolio arguing that as beta fails to consider unsystematic/diversifiable risk. And a risk measure that takes into account total variation of return relative to overall market variation i.e. bearing diversifiable risk and systematic or nondiversifiable risk is representattive one. Alpha is unrelated to market movements and is interpreted as a return attributable to the fund manager's skill (or luck) and a positive alpha is often used as a hallmark for an investor talent. For a given set of data, beta estimation method will affect the consequent value of alpha.

Estimation of beta in CAPM has been criticized on various grounds as well. Statisticians use regression models to minimize the sum of squared errors in the dependent variable only – this is

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because the purpose of regression is to fit a relationship for predicting the dependent variable (rate of return of the investment) for a stated value of the explanatory variable (the market rate of return). While definition of returns, whether continuous (i.e. difference of log price) or discrete returns (ratio of adjusted price diserence or in percentage terms), does not affect the results (Nathaphan), but OLS assumes reliability of measurement of market return in beta estimation. It means that the independent variable (market return) has no measurement error. However, Roll and Ross (1992) claims that with the use of market index as a proxy, there will be an error present (called the errors in variables problem or benchmark error) and it will affect estimate of the slope or beta (QFT). And because of this affect, CAPM is not a testable theory unless we know the exact composition of the market portfolio (Roll's; 1977; QFR). OLS only assumes error in the dependent variable. Elton et al, (2003) proves that if the explanatory variable has a random error with even a zero mean, this will still lead to a slope estimate in the security market line which is too low (downward biased) and thus a too high estimate for the intercept. Statistics also tells us that in simple correlation and regression, unreliable measurement causes relationships to be under-estimated thus increasing the risk of Type II errors. In the case of multiple regression or partial correlation, effect sizes of other variables can be over-estimated if the covariate is not reliably measured, as the full effect of the covariate(s) would not be removed. Thus there is a potential for Type II errors for the variables with poor reliability, and Type I errors for the other variables in the equation. Whilst there are estimation methods for dealing with measurement error in the independent variable, they require knowledge about the variance of the error and this is simply not known. Thus desirable condition for the estimation of beta is that it should allow for measurement error in the variable (i.e. in choice of proxy for market return) and estimation of security market line which allow for error in the explanatory variable (QFT). However, to address the robustness of the estimation based on proxy data (i.e. book value), Reidl (2009) has compared it to the estimation using market value of equity for the derivation OA variable as well as to scale all variables in his beta model. In this specification, OA are computed as the difference between the sum of market value of equity plus book value of liabilities less the sum of the fair value assets. Further, all variables are deflated by the sum of the market value of equity plus book value of liabilities. The results showed the magnitudes of the betas were similar to those of the primary analyses. There is also a question about functional form – linear or non-linear, besides the number of factors to be included. The assumption of constancy of parameters (α & β) which derives equation (5) or beta estimation assuming market efficiency hypothesis (i.e. $\alpha = 0$ and α is not r_f rather is investment alpha) are not very valid as well. In later case, reasonable assumption is that $\alpha \neq 0$ showing that mispricing for a set of traded assets. Various studies (QFT), telling the story of non-constancy of beta, have been provided below. Hirschey (2001) shows that for Dow Jones stocks, the correlation between current year betas and previous year betas is only 0.34. Chawla (2001) reviews the literature on beta stability and uses hypothesis tests to demonstrate instability and refutes the thesis of stability of beta as in such case there will be an Eternal Beta Bible citing beta values to be true for all times. In reality, there is a demand for beta books like Value Line Investment Survey, Bloomberg, Standard and Poor's, Ibbotson Associates and the Risk Measurement Service of the London Business School. Francis (1979) found explicit evidence pinpointing each stock's correlation with the market as the most unstable statistic within beta and concluded that the correlation with the market is the primary cause of changing beta, the standard deviations of individual assets are fairly stable. Fabozzi and Francis (1978) investigated 700 stocks on the New York stock exchange and found that many stocks betas move randomly through time rather than remain stable as the ordinary least squares model presumes. They demonstrate that the partitioning of risk will be confounded with the noise from the shifting beta. As a result it will not be possible to estimate empirically the separate effects of systematic and unsystematic risk. This particular implication undermines too many empirical studies to list here and authors even went on saying that the ordinary least squares regressions used in nearly every instance may be inappropriate (QFT). Many studies confuse beta with relative volatility and

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verbal explanations of beta are often incorrect and give the wrong impression i.e. common interpretations applied to beta in finance are not consistent with the formula used to estimate (Tofallis; 2009). Volatility, often used as a measure of risk, is measured by the standard deviation of the rates of return in finance. In order to compare the volatility of an investment's rates of return with the volatility of the market rates of return we can simply use the volatility ratio or relative volatility. Thus the standard method of estimation – least squares regression – is inconsistent with these interpretations. These interpretations are important because many financial decisions are being made daily by analysts using this interpretation. Thus Standard beta is not the same as relative volatility. If an investment had the same risk (volatility) as the market then its volatility ratio would equal unity, but standard beta would not equal unity. Instead, its beta would equal its correlation with market returns, and hence would always be less than unity (QFT).

Another major concern is the estimation risk in parameter estimates in OLS (Barry and Brown (1984, 1985), Clarkson and Thompson (1990) and Reidl (2009). The estimation risk in OLS estimation is well documented and because of it, especially during the financial crisis period, optimal portfolio is not an optimal investment as intended. Two crucial parameters in an efficient portfolio construction are expected return and variance-covariance matrix. Estimation risk in portfolio formation is caused by treating sample estimates as true parameters. Various studies (all below 'references from Nathaphan (2010) (RFN hereafter)' unless mentioned otherwise) can be divided into various groups. The first group conducted the studies based on historical data ignoring estimation risk. This group includes Markowitz (1952), Sharpe (1964), Kraus and Litzenberger (1976), Kroll, Levy and Markowitz (1987), and Chunhachinda et al. (1997a, b). The second group used various approaches to lessen the estimation risk in univariate time series models by including trend, seasonal, and residual component method, Frequency Based Methods, Autoregressive (AR) and Moving Average (MA) Models and Box-Jenkins Approach. Thirdly, Models capturing the time-variation of beta have been compared in Faff et el. (2000) (QFT). Fourth group focused on the asset pricing approach by incorporating a factor model such as the Capital Asset Pricing Model (CAPM) and/or Arbitrage Pricing Theory (APT) in the portfolio selection process, e.g. Polson and Tew (2000), and Pastor (2000). This group uses a factor model to benchmark the performance of a recommended portfolio. Portfolio optimization is performed based on historical data to estimate two crucial parameters of the model, namely, expected return and variance-covariance matrix. Estimation risk due to treating sample estimates as true parameters had been taken into account in optimal portfolio formation via Bayesian Portfolio Optimization process. Fifthly, Nathaphan (2010) has presented Bayesian Single Index Model (BSIM) in order to undermine the estimation risk and has compared the results with conventional estimation strategies for efficient portfolios i.e. traditional mean-variance (EV), Adjusted Beta (AB), Resampled Efficient Frontier (REF), Capital Asset Pricing Model (CAPM) and Single Index Model (SIM) during two financial crisis periods. The sixth group of studies took estimation risk into account by proposing a Bayesian or resample efficient frontier approach using historical data together with Monte Carlo estimation process; for example the studies of Stein (1962), Kalymon (1993), Barry(1974), Klein and Bawa(1976), Brown (1979), Chen and Brown (1983), Jorion (1986), Horst et al. (2002), Markowtiz and Usmen (2003), and Michaud (2006). Estimation risk does not change the efficient set but will affect the optimal portfolio (Barry; 1974). The effects of estimation risk on the selection of an optimal portfolio from a set of risky assets cause the location, but not the composition, of the efficient frontier to change, assuming that security returns are generated by a stationary multivariate normal distribution (Bawa, Brown, and Klein; 1979), (Klein and Bawa; 1776). Frost and Savarino (1986) studied portfolio selection within a Bayesian framework to deal with estimation risk and stated that using classical mean to estimate expected return and other moments of asset returns leads to suboptimal portfolio choices resulting in a loss of investor utility. Same loss was stated by Jorion (1986, 1991) with

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uncertainty about parameter values if historical average is used as a true parameter estimate. Besides, it validated that the sample mean is an inadmissible estimator by the James-Stein estimator which is derived from the summation of components of a quadratic loss function using a shrinkage function to estimate parameter values and also extended the work of James-Stein to a Bayes-Stein shrinkage mean assuming that variance parameters are known. Britten-Jones (1999) used 20 years of data on 11 country stock indexes to conclude that sampling error in estimates of the weights of global efficient portfolios is as large as when the return vector and variance-covariance matrix are estimated by a traditional approach. In seventh group is a large group of literature that relates equity beta to parameter estimation risk (Barry and Brown; 1984&1985); Clarkson and Thompson (1990); Coles, Loewenstein and Suay (1995) QFR). These studies show that systematic risk decreases the quality of information. Riedl et el (2009) examined the interplay between information risk and the reporting of financial instruments at fair value which was motivated by high-level policy debate regarding the role of fair values in the current economic crisis.

Besides estimation risk, regressioneering (i.e. sub-optimal or even misuse of regression) (Anjum 2014b), and regressgineering (from regression engineering) is also a real issue in many applications of OLS regression. regressgineering is said to be present when researchers in various decscipline try to find the presentable regression by using randomly chosen data samples by including or exclusing certain observations on either front end or back end of data sets in order to get highest R^2, significant t-values or overall model fit (F-stat). Because of these limitations of OLS, it is often recommended that if the goal to use data is to solve problems, then prefer algorithimic modelling culture over data modelling culture – two competing cultures of statistical modeling (Tofallis; 2003). In case of data modelling culture black box may contains linear regression, logistic regression and Cox model etc. while in case of algorithimic modelling culture black box can contain decision tree or nueral nets etc. The later has more diverse set of tools, can be used on large complex data sets and is more accurate and informative alternative for smaller data sets. Despite these advantages of algorithmic models, 98% of all statisticians still have been committed to the almost exclusive use of data models (Breiman in Osbourne; 2009). Regression analysis, a data model, rely upon certain assumptions about the variables used in the analysis and when these assumptions are not met, results may not be trustworthy, resulting in a Type I or Type II error, or over- or under-estimation of significance or effect size(s) (Osborne and Elaine; 2002). Assumptions of the linear regression model are linearity of functional form, fixedness of independent variables, independence of observations, representative sample and proper specification of the model, normality of the residuals or errors, equality of variance of the errors (i.e. no hetroscadasticity or homogeneity of residual variance), no autocorrelation of the errors, no outlier distortions, no multicollinearity and number of observations exceeds the number of coefficients to be estimated (Yaffee (2004). The last two are particular to multiple regression case. The understanding of when violations of assumptions lead to serious biases or are of little consequence is essential to meaningful data analysis (Pedhazur; 1997). Caring for these assumptions helps avoid issues like attenuation due to low reliability, curvi-linearity, and non-normality which often boosts effect sizes, usually a desirable outcome. OLS continues to be by far the most frequently used method even when it is obviously inappropriate. As a result, hundreds if not thousands of regression lines with too-small slopes are being published annually (Riggs et el.; 1978). Likewise, despite various shortcomings mentioned above of standard beta and OLS estimation, the most easiest and popular technique to estimate beta is only OLS.

FINANCIAL SERIES AND DATA OUTLIERS

McAleer (2006) decribes that outliers in the financial data have been present even before the crisis of 2008. He has given the descriptive statistics of synchronous data of returns i.e. data of

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daily prices (in US dollars) measured at 16:00 Greenwich Mean Time (GMT) obtained from DataStream for the period 3 August 1990 to 5 November 2004 for four international stock market indexes i.e. S&P500 (USA), FTSE100 (UK), CAC40 (France), and SMI (Switzerland). Each of the series displays a high degree of skewness and kurtosis and Jarque-Bera test strongly rejects the null hypothesis of normally distributed returns indicating the existence of extreme observations. And the true volatility of returns (defined as proposed in Franses and van Dijk, 1999) of all series is high in early 1990's, has a quiet period from end 1992 to the beginning of 1997, increases dramatically around 1997 (due to Asian financial crises) and persistence of this increase until 2004 (because of 9-11 and Afghanistan and Iraq war affects), says the article. Thus outliers creates a violation of normality assumption (i.e. variables have normal distributions) of OLS. Variables with substantial outliers or of highly skewed or kurtotic variables can increase the probability of Type I and Type II errors by distorting relationships, significance tests and thus the accuracy of estimates but only few regression articles report the statistical testing of these assumptions especially in the literature of social sciences and thus validity of many of these results, conclusions, and assertions are called into question (Osborne et el.; 2001). Under-estimation of true relationship carries two risks. One is that it increases chance of a Type II error for that independent variable (IV) having outliers, and in the case of multiple regression, an increased risk of Type I errors (over-estimation) for other IVs that share variance with IV having outliers. Regressgineering also becomes a real threat when financial data set contains outliers because of the effects of financial crisis on data. While outliers by themselves only distort mean prediction when the sample size is small enough, it is important to gauge the influence of outliers (Yaffee; 2004).

There are various ways to test the assumption of normality – informational and inferential. Visual inspection of data plots, skew, kurtosis and P-P plots are examples of the former and Kolmogorov-Smirnov tests provide inferential statistics on normality. Outliers can also be identified either through visual inspection of histograms or frequency distributions, by converting data to z-scores or tests to look for no serious outlier influence, additive outliers and pulse dummies. Tests includes the plotting of residuals and looking for high leverage of residuals, lists of standardized residuals (SaR), lists of studentized residuals (SuR), Cook's distance or leverage statistics. Outlier diagnostics and detection (ODD) performed through residuals or errors (i.e. predicted value minus the actual value) or ($e_i = \hat{y}_i - y_i$) involves the determination whether e_i is an extreme negative or positive value. We may also plot the residual versus the fitted plot to determine which errors are large, after running the regression. Belsley et al. (1980) recommended ODD through SuR (i.e. the residuals divided by their standard errors without the ith observation) or (= (e_i) / $[\sqrt{s}\,^2{}_i * \sqrt{(1-h_i)}]$ where s_i is standard deviation when ith observation is deleted and h_i is leverage statistics as quoted in Yaffee (2004) (QIY herefater). If [-3.5 < SuR > 3.5], then it is outlier and if SuR < |3.5|, then there are no outliers. SuR are distributed as t-distribution with degree of freedom (df) = n-p-1 where n=no. of observations and p=no. of parameters. They are not quite independent and we can approximately determine if they are statistically significant or not. ODi can also be performed with leverage and Cook's distance. Leverage, also called the Hat diag and is the measure of influence of each observation, is measured by the diagonal components of the hat matrix which comes from the formula for the regression of Y as: $\hat{Y} = X\beta = \grave{X}(\grave{X}\,X)^{-1}$ $\grave{X}Y$ where $\grave{X}(\grave{X}\,X)^{-1}\,\grave{X}$ = the hat matrix = H. Therefore $\hat{Y}$ = HY. The hat matrix transforms Y into the predicted scores and the diagonals of the hat matrix indicate which values will be outliers or not. Leverage is bounded by two limits: 1/n and 1. The closer the leverage is to unity, the more leverage the value has. The trace of the hat matrix = the number of variables in the model and when the leverage > 2p/n then there is high leverage according to Belsley et al. (1980) (QIY). For smaller samples, Vellman and Welsch (1981) suggested that leverage > 3p/n is the criterion (QIY). "Cook's Distance or D" is another popular measure of influence. It's formula is: Cook's-$D_i = [(1/p)*\{hi/(1-hi)\}]*[(e^2{}_i) / [\sqrt{s^2{}_i}*\sqrt{(1-h_i)}]$. Cook and Weisberg (1982) suggested that values of

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Cook's-D that exceeded 50% of the F distribution (df = p, n-p) are large. Also if change in the statistics, that results from deleting the observation is >1.0, then it need to be watched. To find the influential outliers, look for cook-D [> 4/n] or as Belsley suggests use [= 4/(n-p-1)] as a cutoff (QIY).

There are various alternatives to violations of nonnormality assumption. One can run a least absolute deviations regression or a median regression or generalized linear models. Shalit and Yitzhaki (2002) which discussed the instability of OLS estimators of beta blamed on the quadratic loss function which makes extreme observations have a magnified effect and Martin and Simin (2003) focusing on the effect of outliers and observed that the effect is particularly noticeable for small firms. Former study proposed the use of a coefficient to represent the investor's risk aversion and later the use of weighted least squares estimator where the weights are determined by the data (QFT). Removal of univariate and bivariate outliers can reduce the probability of Type I and Type II errors, and improve accuracy of estimates. However, it is not always desirable to remove outliers. Transformations (e.g., square root, log, or inverse), can improve normality but can complicate the interpretation of the results hence this should be used deliberately and in an informed manner (Osborne et el.; 2001). As an alternative solution, after the regressions are run on total dataset and also on divided datasets which were divided at the time of the event of systematic crisis as in Nathaphan (2010), these regressions are ranked based on Relative Grand Total Priority Score (RGTPS) and then added together in order to get RGTPS weighted average.

METHODOLOGY AND DATA DESCRIPTION

The focus of this study is to apply Best Auto-logic AHP-mated (Be-ALAM) regression which uses AHP weights in order to get RSTPS or Best Auto-logic Equally-mated (B-ALEM) regression which uses equal weights (Anjum; 2014d) derived from regression ranking based on FAIZ approach (also called FAIZ regression) used in Anjum (2014b). FAIZ regression is a pioneer framework to rank various regressions based on RGTPS. Be-ALAM (B-ALEM) regression is based on weights derived from Saaty's Analytical Hirarechy Process (AHP) using pair-wise comparison (equal treatment) for all criteria rather than from fuzzy AHP as in Anjum (2014b). Both Be-ALAM and B-ALEM regressions have used five categories of criteria – assumptions (AS), statistical inference (SI), overall effectiveness (OE), Robustness (RO) and quality considerations (QC) to rank the regressions. Scoring rules for both Be-ALAM and B-ALEM are same. The minmax function is used to get the score for SI if t-value (TV) is between |2| & |1.5|] and the score is zero (one) if TV is less than (equal to) |1.5| (|2|). If p-value (PV) is between 90% and 99%, then minmax function has been used to get the score for OE and score is zero if PV is less than 90%. For RO, which is represented by coefficient of validation data regression (VDR) minus coefficient of estimated regression, VDR for SP1 & TP is chosen SP2 and VDR for SP2 is TP. As the smaller RO is better, we have used maxmin (MM) to keep it in ascending order. Finally compute the "average RO" i.e. average of maxmin of α-RO and β-ROs for each alternative regression. This means that RO of Alpha of TP regression is average of Alpha of SP1 regression and Alpha of SP2 regression minus Alpha of TP regression. As QC contains two parameters i.e. sign of coefficients (SOQ) and magnitude of coefficient (MOC) and SOC and MOC for α and β sum to one. For example if SOC for β got the score of 10/10 i.e. 1 and MOC for β is 9/10 i.e. 0.9 then the total score for QC (β) = [½ (1) + ½ (0.9)] i.e. 0.95. Finally because no results for the tests of assumptions were provided in Nathaphan (2010), AS was assumed to same for all in each set of regressions. . It should be noted, however, that it does not matter whether you give AS criteria a score of zero, one or any other random number, it will scale up each alternative by same number thus will have no effect on the overall ranking of alternatives. This is because it will be same score for each alternative.

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This article has built an Aggregated Zoom AHP-mated (AZAM) regression which will be used for the estimation of Beta. This AZAM regression may provide a solution to the systematic risk outliers that influence the estimation of beta with OLS. Results for α & β coefficients from three different regressions (alternatives) for nineteen different countries based on three different datasets covering three time periods has been considered from Nathaphan (2010) and have to be used in this study as inputs to get our Be-ALAM and AZAM regressions. The nineteen emerging economies covered are Argentina (AR), Brazil (BZ), Chile (CL), China (CH), Columbia(CO), Hungary (HY), India (IA), Indonesia (IS), Malaysia (ML), Mexico (MX), Pakistan (PK), Peru (PU), the Philippines (PH), Poland (PO), Russian Federation (RF), South Africa (SA), Taiwan (TW), Thailand (TH), and Turkey (TK). One of the time periods covered for regressions for each country include a timeframe of 1995 to 2008 called total period or TP which covers various global financial crises including the events like Asian financial crisis in 1997, Long-Term Capital Management (LTCM) crisis in Japan and Russia in 1998, IT bubble burst and dotcom crisis of 2000, bankruptcy scandal of ENRON in late 2001 and bond market crisis in 2003. But it was the US subprime crisis of 2008 that can be said to have created data outliers in financial databases. Thus TP data has been further divided into two sub periods, 1995-2001 (called sub period 1 or SP1) and 2002-2008 (called sub period 2 or SP2). Data used in this study are monthly index returns of 19 emerging markets adjusted for dividend and emerging market price index list of countries in emerging markets based on FTSE emerging market list. Data was obtained from Data Stream under DSM mnemonic TOTMKEK in U.S. dollar unit and quotations of each market index are based on U.S. dollar.

In short, each criterion has been provided a weight obtained with the help of Saaty's Analytical Hierarchy Process (AHP) obtained from pair-wise comparison (equal treatment) of criteria for Be-ALAM (B-ALEM) regression. Besides this weighting system for all five criteriae for both Be-ALAM and B-ALEM regressions, we also need to build a scoring system based on certain rules for these five criteria. These rules will be able to assign separate scores to each alternative. The simplified description of the rules to assign score for these five criteria has been provided above. The weights obtained from crisp AHP (equal treatment) for AS, SI, OE, RO, QC are 0.4584 (0.2), 0.264 (0.2), 0.1523 (0.2), 0.0973 (0.2) and 0.0281 (0.2) respectively. The scores attained by each criteria from these rules provide values for each alternative in relative form and has been brought within a range between 0 and 1 to tackle the issue of commensurability among criteria by using Kramer (2008) style step wise utility function (SWUF) that uses minmax function (Anjum; 2014a). These scores are determined for each alternative i.e. for each of three regressions for each country. Finally relative score of each five criteria attained by each alternative (regressions in our case) have been multiplied with AHP weights (AW) or equal weights (EW) of each five criteria to get the Weighted Total Score for each criterion ($WTS_{criteriae}$). In order to rank the alternative regressions (three regressions for each country in our case), we need to add the weighted total scores for all five criteria in order to get Grand Total Priority Score (GTPS) for each alternative i.e. $GTPS = \sum(WTS_{AS} + WTS_{SI} + WTS_{OE} + WTS_{RO} + WTS_{QC})$. The total priority or GTPS for each alternative is the degree to which that alternative fits all the criteria, sub criteria, and scenarios. This final step is achieved through decision matrix. Finally, these GTPS for all alternatives are normalized in order to get relative GTPS (or RGTPS) and an alternative (i.e. regression here) with the highest RGTPS is superior to the alternative with lower RGTPS. The RGTPS attained by each regression for each country has been provided in table 1.

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Table 1: RGTPS for TP, SP1 and SP2 Regressions for 19 Emerging Economies							
Countries	TP	SP1	SP2	Countries	TP	SP1	SP2
Argentina	0.4312	0.3323	0.2365	Pakistan	0.3440	0.3280	0.3280
Brazil	0.3592	0.4002	0.2406	Peru	0.3473	0.2824	0.3702
Chile	0.2035	0.3982	0.3982	Philippines	0.3086	0.3605	0.3309
China	0.4192	0.2521	0.3287	Poland	0.3031	0.3725	0.3244
Columbia	0.2998	0.2561	0.4441	Russian Federation	0.3561	0.4084	0.2355
Hungary	0.3808	0.4462	0.1729				
India	0.3570	0.2983	0.3447	S. Africa	0.3611	0.4052	0.2337
Indonesia	0.3434	0.2841	0.3725	Taiwan	0.2935	0.3388	0.3677
Malaysia	0.3134	0.3852	0.3013	Thailand	0.3065	0.3595	0.3341
Mexico	0.3379	0.3821	0.2800	Turkey	0.3564	0.4082	0.2354

From these RGTPS, we can easily verify that which one of the three regressions for each country have received the highest score. The regression with hightest RGTPS obtained from AHP (equal) weights for each country is our Be-ALAM (B-ALEM) regression. For each of the ninteen emerging economies, second column of Table 2 shows that which regression (from TP, SP1 and SP2 Regressions) got the highest RGTPS score based on equal weights (EW) and/or AHP weights (AW). The idea of dividing the total period dataset into two sub period was based on the reason that if we will run regression on total data only then, because of systematic risk outliers in the financial data (for US sub-prime crisis of 2008), the accuracy of regression coefficients (α & β) may be distorted because of the probability of Type I and Type II errors and/or distortions in the relationships and significance tests (Osborne (2001)). The concept of AZAM regression is very simple but a powerful one. AZAM regression is obtained after taking the weighted average of alternatives (i.e. regressions for TP, SP1 and SP2 for each of 19 countries) where weights for each regression are the RGTPS of the respective regression. In other words, each regression, based on TP, SP1 or SP2, is multiplied with their respective RGTPS and then all these three regressions will be added together and the resulting regression is called Aggregated Zoom AHP-mated (AZAM) regression. The AZAM regression obtained from regressions ranked using AHP (equal) weights is called AW-AZAM (EW-AZAM). Table 2 shows the values of α from B-ALEM and/or Be-ALAM (third column), EW-AZAM (fourth column) and AW-AZAM (fifth column) and values of β from B-ALEM and/or Be-ALAM (sixth column), EW-AZAM (seventh column) and AW-AZAM (eighth column), for each of the ninteen emerging economies.

RESULTS AND CONCLUSION

Basic regression results for countries (mentioned above) have been provided in Table 1(a) of Nathaphan (2010). Empirical results indicate that all emerging markets exhibit nonzero alpha with positive beta coefficient as postulated by theory. Only few countries with nonzero alpha are statistically significantly different from zero while all beta coefficients are statistically significantly different from zero. This could be interpreted in a way that emerging markets' risk and expected return relationship conform to modern portfolio theory and there is mispricing in some emerging countries. Fund managers can insert their own belief in determining countries with mispricing and recognize the abnormal return from such portfolio formation strategy. Moreover, Nathaphan (2010) shows that the average returns in each sub period are not significantly different. Based on TP (SP1) [SP2], average monthly index returns range from -0.44% to 3.35% (-1.15% to 5.79%) [-0.72% to 1.95%]. Unlike average index return, standard deviations or risk levels for each country are significantly different and larger than average return. Based on total sample index return, standard deviations for TP (SP1) [SP2] range from 4.79% to 15.59% (5.67% to 19.33%) [3.72% to 10.37%]. The information ratio (IR) provided indicates that

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the mispricing of each country is large as each value differs from zero considerably and spread of variation of IR was wider in the second sub period reflecting the recent sub prime financial crisis.

Table 2: Name of top RGTPS Regression(s) in the first column and Values of α & β for B-ALEM/Be-Alam, EW-AZAM & AW-AZAM Regressions in other columns

	B-ALEM / Be-ALAM	α for B-ALEM/Be-Alam	α for EW-AZAM	α for AW-AZAM	β for B-ALEM/ Be-Alam	β for EW-AZAM	β for AW-AZAM
TK	SP1	0.0664	0.0394	0.0394	1.2406	1.1033	1.1027
RF	SP1	0.0365	0.0223	0.0223	1.6427	1.425	1.4255
HY	SP1	0.0253	0.0130	0.0138	0.8654	0.8177	0.8198
MX	SP1	0.0142	0.0102	0.01	0.8014	0.7058	0.7028
BZ	SP1	0.0129	0.0099	0.0098	0.9865	0.904	0.9035
CH	TP	0.0078	0.0073	0.0073	0.9028	0.9104	0.9067
SA	SP1	0.0108	0.0075	0.0075	0.7489	0.6849	0.6852
IA	TP	0.0056	0.0052	0.0051	0.7896	0.803	0.799
PO	SP1	0.0157	0.0063	0.0061	0.7206	0.7172	0.7171
IS	SP2	0.0059	0.0049	0.0048	0.8155	0.7886	0.7874
CO	SP2	0.0163	0.0065	0.0066	0.5128	0.4414	0.4425
PU	SP2	0.0058	0.0041	0.004	0.5697	0.516	0.5136
PK	TP	0.0036	0.0033	0.0033	0.4629	0.4487	0.4486
AR	TP	0.0033	0.0055	0.0053	0.7899	0.8206	0.8135
CL	SP1 & SP2	-0.0012 &0.004	0.0011	0.0011	.5974 & 0.3268	0.4616	0.4617
ML	SP1	0.00279	-.0001	-2E-04	0.9076	0.6961	0.6875
TW	SP1/ SP2	0.007 / -0.0115	-0.0019	-0.003	1.0079/ 0.6949	0.8549	0.8403
PH	SP1	-0.00183	-0.0017	-0.002	0.8578	0.6437	0.6409
TH	SP1/ SP2	-0.00394 / -0.00248	-0.0036	-0.004	1.1006 / 0.8109	0.9553	0.9535

Note: 1. If two entries in cells are separated by (/) then first entry is for EW and second entry after (/) is for AW. 2. If there is an ampercent (&) between two entries, it means that there was a tie between both entries (i.e. SP1=SP2) and thus both are equally an answer for both EW and AW (e.g. SP1 & SP2). 3. If only one entry is provided, it shows that this one entry is only answer to both EW & AW.

Our results are shown in table 2 in detail where first column provides the names of the countries and second column shows that, for each country which of the three alternatives came up with the highest score based on Be-ALAM and B-ALEM. If only one entry is provided in the column, it shows that the same alternative got best scores both in Be-ALAM and B-ALEM cases. If two entries in cells are separated by backslash (/) then first entry has achieved the highest score for B-ALEM and second entry for Be-ALAM. And if there is an ampercent (&) between two entries, it means that those two alternatives have got the same scores (which was the highest one) for both Be-ALAM and B-ALEM cases. The third and sixth columns show the values for alpha and beta of the alternative with the highest score in each country case. And the fourth and seventh columns show the results of the alpha and beta of the AZAM regression obtained from B-ALEM while the the fifth and eighth columns show the results of the alpha and beta of the AZAM regression

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obtained from Be-ALAM regressions.

If we compare the values for the parameters like alpha and beta for different alternatives of various countries from table 1(a) of Nathaphan (2010), we will notice that values of the alpha are the highest for the Be-ALAM and B-ALEM regressions for all economies except AR and TW (excluding PH and TH for simplicity). The values of the beta are also the highest for the Be-ALAM and B-ALEM regressions for all economies except CH, IA, PK, AR and CL whereas TW has highest for Be-ALAM and TH has the highest of B-ALEM (this has been mentioned here because in only TW and TH cases, different alternatives have achieved the highest scores based on Be-ALAM and B-ALEM). The results for the AZAM regression based on both Be-ALAM and B-ALEM approaches has the values of alpha and beta reduced and are just a little higher (in most or all cases) than the second highest values of these parameters in original equations for each economy. Now this is an improvement and is considered as leaning towards the estimator presented by Tofallis (2008) which is called as β^* [$=\beta/\rho$] where ρ is correlation between universal set and the relevant subset i.e. market versus asset. The idea of β^* originated from the proposition by Booth and Smith (1985) (QFT) that estimates from OLS regression and reverse regression are the bounds on the true value of beta. For the usual case of positive correlation between market and the investment, we have the standard beta giving the lowest value and the reverse regression the highest i.e. $\beta_{OLS} \leq \beta^* \leq \beta_{reverse}$ (Tofallis; 2008). Lowering the β_{OLS} (i.e. by using β_{AZAM} instead) will help to pull the average (i.e. β^*) lower and thus more towards real relative risk, instead of exaggerated one. Last but not the least, the Be-ALAM or B-ALEM ranking approaches can also be used for reverse regression which may also lower the upper bound (i.e. $\beta_{reverse}$) thus lowering the measure of relative risk (i.e. β^*) even further. This also means that with the lower value of relevant beta, will increase discount rates by increasing risk premiums which seems more relevant especially because of various non-quantifiable or at least poorly quantifiable (often through the use of proxies with less than 100% correlation to real variable) features of some (if not various) segements of most of the developing economies.

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