A Production Function of Education: Interdependency between Lecturing and Practicing

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ABSTRACT

In recent years, scholars have argued that traditional learning no longer achieves its goals because learning time is used inefficiently. The present goal is to demonstrate a way to more effectively utilize learning time in order to maximize knowledge. The model suggests a production function of education that includes interdependency between lecturing and practicing hours and which should be considered when the goal is optimal time allocation between them. This approach may improve the learning performances of various academic institutions that very often struggle with the question of how to optimally implement the components of academic activities. One may assume that the effectiveness of lecturing time positively impacts practice time and exercise hours provided by teaching assistants. It will also affect the length of each activity and the rotation between lecturing and practicing. The present approach demonstrates several production functions of education with different divisions of time between lecturing and practicing.

Keywords: Lecturing, Practicing, Utilization, Learning Cycles

1. INTRODUCTION

1.1. Optimal Allocation of Learning-Related Resources

In recent years, researchers have claimed that traditional learning no longer achieves its goals. These critics argue that a 50-minute lecture does not create deep and lasting student comprehension and is a relatively inefficient pedagogical tool for promoting conceptual learning (Eison, 2010; Knight and Wood, 2005; Smith et al., 2009). In order for real learning to occur, they claim that the learner needs to be actively involved not only in the doing but also in thinking about the material being learned (Eison, 2010). This approach, often called "Active Learning", seeks to make the student fully engaged...
in the learning experience by using different tools to enhance interest and involvement. These tools may include working in groups, using audience response systems, expressing ideas in writing, exploring attitudes, "giving and receiving feedback and reflecting upon the learning process" (Eison, 2010, p. 1).

Studies which have examined the contribution of these strategies have often found them to promote student learning. For example, Knight and Wood (2005) applied some of these methods in a Developmental Biology course and found learning performance to improve just by changing to a more interactive classroom format. Nonetheless, integrating active learning in the classroom is limited by a variety of factors such as length of a lesson, pre-class preparation, class size, lack of equipment, and lack of cooperation from students (Eison, 2010). All of these factors are dependent upon the budget of a given institution. Academic or educational institutions, similar to other institutions that wish to survive, try to minimize service costs. This basic principle has led to the current trend in educational economics of trying to find the equity point of cost-effectiveness, between output and input costs. Extensive research attempts to delineate the factors that can both save costs and promote learning. Integration of technology, larger class size, shorter class time, and higher student-teacher ratio are some of the factors which have been widely investigated in relation to student learning performance.

We ask how scarce resources like funding or equipment may be most effectively utilized. Do we apply our arguments to efficient time management for teaching a course and providing maximum knowledge to students? How should we divide our time or our budget among frontal lecturing, long-distance learning via devices, etc.? Within a limited time constraint we have to decide upon the length and frequency of each class - 3 hours a day, 3 hours a week, or perhaps 3 one-hour classes. We need to determine the hours for laboratory practice, for use of PowerPoint or other applications to watch and study cases, and for practice with old exams. Additional considerations are the time between class and practice, the actual classroom lecture time, and the practice time provided by a graduate student, tutoring service, etc.

When we examine the practicality of decisions made by the authorities, we may sometimes wonder whether they are guided by a decision planner and maker or whether decisions are perhaps made in haste. In many universities, in which decisions are changed too often, a decision may even appear to be made arbitrarily and lacking sufficient consideration. Our experience in different universities in the United States and in other western countries indicates the use of different combinations of learning methods. The fundamental and most popular economics course in most academic institutions is called Introductory Microeconomics. In different institutions the ratio varies between lecturing hours provided by a professor and practicing by teaching assistants or PowerPoint, Blackboard, Canvas or other applications. We also find that very often the structure is changed after several years. Is this an arbitrary decision? Is it a measured and well-established decision due to new teaching material or new teaching technology? What is the rationale behind it? Is it due to budget constraints or other factors that arise? Does the decision enable the system to indeed use scarce resources in the most efficient way? Our model provides further explanation.
Moreover, one may wonder to what extent decision makers in academic institutions are aware of the interdependency between practicing and lecturing. The positive relationship between them indicates the need to evaluate the optimal length of each activity and number of cycles that most efficiently utilizes financial and time resources.

The preceding discussion can be related to recent innovative ideas regarding the importance of idle time. Brain research scientists are aware that idle time may significantly and positively affect brain activity during its active phase. They find that more idle time affects the use of the brain for innovations at the workplace. The allocation between idle and active time is an important issue to be considered. Based on the concept described above we suggest a "new" production function that includes intertemporal external effects. It departs from several other traditional production functions of education, some of which are considered by Hanushek (1986) who summarizes the results of 147 estimated production functions. In all of them, the variables that are measured use very specific bundles of inputs of influencing variables affecting levels of production performance. However, we have not found a production function that contains both traditional lecturing by professors as well as hours of practicing through review exercises, work in the laboratory, or classroom instruction provided by teaching assistants. Moreover, to the best of our knowledge, the "intertemporal effect" has not been considered in those 147 functions used during the last three decades. In a later paper of Hanushek (2003) we also find that student performance is a dependent variable that is influenced by independent variables such as class size (see Lazear, 2001) or teacher educational background and experience, etc.

Our production function of education includes another element that we have not found in the literature and which we refer to as the "Intertemporal Effect." Consider, for example, the efficiency of a teacher’s lecturing. It is also affected positively by practice time, laboratory hours in life sciences, or recitation and exercise hours provided by assistants, etc. This interaction should be considered in the analysis of the length of each activity and the rotation between lecturing and practicing.

The optimal combination of activities enables them to influence one another. (In our case, practice time positively affects the added value of lecturing). The optimal combination of activities should be considered in terms of the number of cycles of combined activities, and the length of time between the activities within each cycle. This is the purpose of our paper.

2. LITERATURE REVIEW

The importance of service quality in higher education has been discussed from different perspectives during recent decades (Hill et al., 2003; Abdullah, 2006; Voss et al., 2007; Jain et al., 2010; Nadiri et al., 2009; Palli and Rajasekhar, 2012; Sharabi, 2013). The papers referenced above follow Marshall (1998) who presents the debate regarding how service quality should be improved in higher education. Ronald and Amelia (2015) evaluate and determine the dimensions of service quality in higher education and formulate strategies to improve them. The present paper considers some of the aspects
presented by Ronald and Amelia (2015). It discusses the concept of utilization of learning time in order to maximize knowledge, improve learning performance, and optimally implement the components of academic activities at a given time. The present paper also demonstrates the best combination of lecturing and practicing hours with the goal of optimal time allocation among them.

Similar concerns regarding the allocation of learning and practicing time in music education are presented by Harald (2015). Practice plays a major part in the lives of music performance students. A survey at the Norwegian Academy of Music (NMH) found that approximately 40% of new students claimed they had been given little or no guidance concerning how to practice. The Teaching of Practicing project that was carried out under the auspices of the Centre of Excellence in Music Performance Education (CEMPE) at the Norwegian Academy of Music (NMH) was intended to ensure effective teaching of practice strategies.

Downs (2014) used the study of geography to examine the conditions, especially the time requirements, for developing geographic literacy and expertise. How much time and practice does one need to become literate or expert in geography? At what age does someone begin to learn geography? What are the cumulative hours spent learning geography and the nature, sequencing, and duration of the learning activities? What are the relationships among (a) the total time spent practicing and the level of expertise attained; (b) the nature of activities that are practiced and the level and type of expertise that are developed; and (c) the total time spent, the practice activities, and the stages through which a person progresses? How much time does the typical graduate from high school, college, or graduate programs spend doing geography?" (Downs, 2014, p. 189).

Kopka et al., (2016) discuss utilization of learning time by integrating academic activities such as lectures with practical work experience. These needs arise from the requirements of employers to hire students who have professional work experience (Scott, 2013). The adoption of an internship course by American universities is intended to enable students to make the “transition from school to work” (Olson, 2014).

All the examples mentioned above concentrate on the basic concept that when looking for efficient methods to learn and educate, one almost always faces a combination of learning theoretical concepts with practicing due to the significant interdependency between them. The question concerns how to determine the length of the cycle and the time allocation between activities in a way that may lead to the most efficient utilization of time. The present study develops a model that seeks to deal with this question.

Another consideration regarding efficient learning is the use of technology that has generally been thought to produce better learning performance. For instance, Sugant and Anvekar (2014) show that Digital Learning Solutions (DLS) contribute to effective knowledge delivery by teachers and enhance the quality of teaching. In another paper Sugant and Anvekar (2016) find that with regard to Digital Learning Solutions (DLS), information quality is positively associated with service quality. However, the assumption that the use of technology produces better learning performance is refuted...
by Brown and Liedholm (2002) who find that students in live classes outperformed their peers in virtual and hybrid courses. This gap was maintained even when students in a virtual class had higher ACT (American College Testing) scores or grade averages. By comparing the performance of students enrolled in a microeconomics course taught in one of three modes of instruction, Brown and Liedholm (2002) concluded that an entirely on-line course would most likely result in lower examination scores, especially in relation to more complex learning materials. In this case, investing in technology (i.e. adding inputs) seems to be ineffective since it does not create the desired value (i.e. outputs).

Higher education experienced a paradigm shift in which lecturers critically evaluate the effectiveness of the traditional class lecture (Johnson, et al., 1991a, b; Laurillard, 1993). Many professors had been growing dissatisfied with the student mastery level of key concepts upon their completion of introductory courses, and consequently, began to examine the pedagogical techniques implemented in the classroom. Throughout academic institutions, a variety of innovative, learner-centered strategies began to replace the traditional lecture. Just-in-Time Teaching (Novak et al., 1999) also referred to herein as “JiTT”, was among the practices introduced to interest university students with diverse learning styles and a variety of academic and social backgrounds. Abreu and Knouse (2014) contributed to the growing discussion regarding instruction in upper-level foreign language courses. They present how the pedagogical technique of Just-in-Time Teaching (Novak et al., 1999), which has been implemented in academic disciplines, could be integrated in foreign language classes.

Chantoem and Rattanavich (2016) compared the English language achievements of vocational students and showed that use of JiTT through web technologies is effective in teaching vocational students. The process of using JiTT through web technologies allows students to interact with each other, experience learning, and compose materials in a variety of ways. Students are engaged in learning with JiTT through web technologies because they can enjoy experiential learning. They can formulate new ideas, evaluate them, recognize problems, raise questions, and learn new vocabulary. They can also consider the purpose of writing their texts. Students with positive attitudes toward this method are more likely to learn than students who only experience conventional teaching techniques. The use of JiTT with web technologies can be a helpful tool for connecting students with educational resources and facilitating participation in collaborative learning with communities outside the classroom.

Another type of resource that can potentially save costs and create value for students is the length of a course. The question has always been whether a larger number of shorter class sessions are equivalent to fewer but longer class sessions or vice versa.

The answers to this question in the literature are often surprising and contradictory. For instance, Austin and Gustafson (2006) studied the link between course length and student learning and found that three, four and eight-week semesters significantly increased student performance over that achieved during a traditional 16-week semester. Moreover, a semester lasting four weeks was found to provide the optimal...
student performance. According to the researchers, improved grades reflected greater learning.

Contrary to these findings, Carrington (2010) also examined the effect of different scheduling formats on student performance and found that students in intensive course schedules perform no differently than students in less intensive course.

Reardon et al. (2008) reported similar findings after comparing student performance in three class formats: 1 hour three times a week, 1.5 hours twice a week or 3 hours once a week. The results indicated that class format had only a marginal impact on student grades. Student performance in the intensive class format was the lowest and student performance in the short format was only slightly better. The study also assessed student preferences for class format and found that regardless of their major, students prefer the moderate class format (1.5 hours twice a week) to the other two options. According to the authors, students may prefer this format because it is less fatiguing and allows them to retain some scheduling flexibility.

Other studies focused on class size as a learning-related resource that can be redistributed. Although class size may reduce cost per student, it may have negative effects on student performance (Bandiera et al., 2010). Finding the optimum class size has been the subject of many studies. However, research literature gives a mixed impression, primarily since the effects of class size are hard to detect (Lazear, 2001). Hanushek (1989; 1997; 2003), after studying the effect of class size, concluded that there is little or no association between performance and class size. In his view, the production function of education has little to do with student performance (Hanushek, 2003). Moreover, Hanushek (1986) claims that teacher-student ratios as well as teacher education and teacher experience are not determining factors in a student's success.

In contrast, Krueger (2002) reanalyzed Hanushek’s data, criticized his selection criteria, and came to the opposite conclusion, "When the various studies in Hanushek’s sample are accorded equal weight, class size is systematically related to student performance, even using Hanushek’s classification of the estimates, which in some cases appears to be problematic." (p.6). Krueger's findings concur with more recent studies which found smaller class size associated with better student achievement (Breton, 2014; Fredriksson et al., 2013; Shin and Raudenbush, 2011; Watson et al., 2013). Other studies demonstrated that 25 students in a class is an optimum point of cost-efficiency (Lazear, 2001). Further evidence in contrast to Hanushek (1986) findings also suggests that the professional skill of the teacher plays an important role in the academic outcome (Brühwiler and Blatchford, 2011).

When drafting a formula for promoting learning, one must take into consideration the unique characteristics of the teaching profession. At the tertiary level, there are inherent differences between the "hard" disciplines (in science and technology) and the "soft" ones (in the humanities). These differences include personal interaction and preparation time as well as research supervision and undergraduate teaching loads. Laboratories in science and technology and tutorials in the humanities are prime examples of the differences in teaching methods (Neumann, 2001). A Norwegian study found that
academics spent an average of 21.2 hours a week preparing lessons and teaching (Smeyb, 1996 as cited In: Neumann, 2001). Academics in the “soft” disciplines spent more time on teaching than their colleagues in the “hard” applied disciplines.

In summary, technology, class size, class length and active learning strategies as well as the teaching culture of various disciplines are all potential resources that can either promote or hold back student performance.

3. THE THEORETICAL MODEL

3.1. Definitions, Notations and Assumptions of the Model

The following model assumes a very specific microeconomic production function in which the dependent variable is defined as the value of knowledge. It is dependent upon and positively affected by the two independent variables of lecturing hours and practicing hours. All other variables that we may find as factors influencing education and knowledge are assumed to be constant, but the model considers how a given total time, $T$, is devoted to lecturing and practicing. The primary innovation of our work that creates a more sophisticated solution arises from the intertemporal, interdependent, and positive effect of lecturing and practicing.

In the model, each course starts with an initial lecture followed by a given period of time, $T$, which is divided into $n$ identical cycles of studying. Each such cycle includes lecturing time and practicing time. The production function of the total value of studying activity is $n$ times the accumulated value of studying activity in each cycle.

The following assumptions are made:
- The marginal value of practicing time is constant.
- The marginal value of lecturing time depends negatively on the lecturing time and positively on the preceding practicing time. Thus, in addition to the apparent contribution of practicing, it also enhances the quality of subsequent lecturing time. However, the positive effect of the preceding practicing time is of a diminishing rate.

Using mathematical notations, the Value of Marginal Productivity of Lecturing, $VMP_{t_L}$, is a function that includes the two variables of practicing time, $t_P$, and lecturing time $t_L$:

$$VMP_{t_L} = f(t_P, t_L)$$

Based on the preceding assumptions we conclude that $VMP_{t_L}$ satisfies the following:

(a) $\frac{\partial f}{\partial t_P} > 0$
(b) $\frac{\partial^2 f}{\partial t_P^2} < 0$
(c) $\frac{\partial f}{\partial t_L} < 0$

The important concept of interdependency is based on the presumption that practicing time is independent of lecturing time, but that lecturing time is more productive due to previous practicing. Therefore, the benefits of practice can be divided into two parts. It is assumed that the direct marginal benefit of practicing time $t_P$ is constant. It is further assumed that there is an indirect positive benefit of practicing time in the values of marginal productivity of lecturing. This means that $t_P$ is "supportive" to $t_L$.
The course is structured as follows. There are \( n \) identical cycles of studying, each of which starts with practicing time, \( t_p \), followed by lecturing time \( t_L \). The total length of the course is \( T \), while the length of each cycle is

\[
\hat{t} = t_p + t_L
\]

and thus the number of cycles is

\[
 n = \frac{T}{\hat{t}}.
\]

At each cycle, the accumulated value of productivity is the sum of two values:

(a) The value accumulated from practicing time, \( t_p \). This accumulated value equals \( \varepsilon \cdot t_p \), where \( \varepsilon \) represents the constant marginal value of productivity from each additional time unit of practicing.

(b) The value accumulated from lecturing time, \( t_L \). This value is given by

\[
V(t_p, t_L) = \left( \int_{0}^{\hat{t}-t_p} f(t_p, t_L) \, dt_L \right)
\]

The value of each cycle is \( V(t_p, t_L) + \varepsilon t_p \).

Based on our assumptions, the function of total value of studying activity \( V \) is given by:

\[
V(t_p, \hat{t}) = \frac{T}{\hat{t}} \left( \int_{0}^{\hat{t}-t_p} f(t_p, t_L) \, dt_L + \varepsilon t_p \right)
\]

where the right term in parentheses is defined as \( v \) representing the accumulated value of productivity at each cycle.

Based on the Leibnitz integral rule, a general analytical optimum is the solution of the two equations obtained from the partial derivatives:

\[
\frac{\partial V}{\partial \hat{t}} = -\frac{T}{\hat{t}^2} \left[ \int_{0}^{\hat{t}-t_p} f(t_p, t_L) \, dt_L + \varepsilon t_p \right] + \frac{T}{\hat{t}} f(t_p, \hat{t} - t_p) = 0
\]

\[
\frac{\partial V}{\partial t_p} = \frac{T}{\hat{t}} \left[ \int_{0}^{\hat{t}-t_p} \frac{\partial f}{\partial t_p}(t_p, t_L) \, dt_L - f(t_p, \hat{t} - t_p) + \varepsilon \right] = 0
\]

Clearly, we are subject to the following constraint:

\[
0 \leq t_p \leq \hat{t} \leq T
\]

In general, we cannot guarantee an analytical maximum satisfying this constraint. It depends upon the behavior of VMP\(_L\) as shown in the following two models. Our innovation in this study is formulating the impact of the interdependency between practicing and lecturing time. Accordingly, the goal is to find values of practicing time and lecturing time, and thus the number of cycles that maximize total productivity of the course during \( T \). These values depend on the parameters of the model. For some parameters, it will be preferable to divide the total length of the course into many short cycles, while for other parameters a long practicing time followed by one period of lecturing time will maximize the total productivity of the course.

3.1.1. Two Models

3.1.1.1. Model 1: The General Case of the Education Function

Based on the division between practicing and lecturing we assume a function \( f(t_p, t_L) \) describing the Value of Marginal Productivity of Learning at each cycle, that is denoted by VMP\(_L\). This function depends on \( t_p \), practicing time and on \( t_L \), lecturing time. The accumulated value of productivity at each cycle is denoted by \( v \). Since the number of cycles, \( n \), is equal to \( \frac{T}{t_p + t_L} \), the total value of productivity \( V \):
The function $V$ should be maximized with respect to the two decision variables, $t_p$ and $t_L$. The solution for maximizing $V$ depends on the specific function $VMP_{t_L}$.

The discussion, below, demonstrates some examples that introduce analytical or simulative solutions. Our discussion begins by focusing on a specific marginal productivity function $VMP_{t_L}$:

$$VMP_{t_L} = A t_p^\alpha - B t_L^\beta.$$  

This function has two parameters: The parameter $0 < \alpha < 1$ describes the positive effect of practicing time on the marginal value of lecturing time; and the parameter $\beta$ is a measure of the rate in which the productivity of lecturing time is diminishing during the lecturing time. The effect of varying the parameters on the course structure is summarized below:

**Proposition A:** The higher the rate of decreasing productivity of lecturing over time, (that is, when the values of the parameter $\beta$ are large), the shorter the length of each cycle. As a result, the number of cycles increases when $\beta$ increases and the total value of lecturing at each cycle decreases.

**Proposition B:** When $\beta$ increases, both practicing time $t_p$, and lecturing time, $t_L$, decrease. However, $t_p$ diminishes faster than $t_L$. Thus, when $\beta$ increases, a larger portion of each short session of $\bar{t}$ is devoted to lecturing and a smaller portion is devoted to practicing. (See Figure 1.)

Sometimes practicing time can become greater than lecturing time while sometimes the opposite is the case. For example, in an academic microbiology course, for each hour of lecturing the student might practice at the laboratory for several hours. In contrast,
in an abstract course such as mathematical physics or microeconomics, the time is primarily devoted to lecturing and less to practicing.

In this specific function, for example, when α = 0.5, practicing time is indeed greater than lecturing time for all chosen values of β. On the other hand, when β increases, the proportion between lecturing time and practicing time changes in favor of relatively more lecturing time.

As stated, the total course length, \( T \), is determined in advance. It should be emphasized, however, that in an extreme possibility the solution for \( \hat{t} \) may exceed \( T \), violating the constraint \( \hat{t} \leq T \). This case has a unique time structure for the course. The time \( T \) is used as one "long" practicing time followed by one session of lecturing. The length of the periods of practicing and lecturing will be determined to maximize the productivity in one cycle. Whether several identical educational cycles or only one cycle may be obtained, depends upon \( T \) and \( \alpha \). This case is described below.

3.1.1.2. Model 2: The Case of a Linear Diminishing Production Function of Education

In this model, it is assumed that the marginal lecturing function \( VMP_{t_L} \) is a diminishing function of lecturing time by a constant rate as follows:

\[
VMP_{t_L} = At_p^\alpha - Bt_L
\]

Proposition C, below, is derived from the preceding marginal productivity of lecturing in (9).

**Proposition C:** When the marginal value of the learning function is linear in learning time, it is most time effective to use lecturing and practicing time in one cycle in which lecturing follows a long practice time.

The development of the formulas for this model leads to the conclusion that the total productivity is an increasing function of practicing time \( t_p \) with no analytical maximum. That leads to extreme allocation of time, with only one cycle. In this case, maximal possible \( t_p \) is optimal such that the sum of practicing time and lecturing time will not exceed the total length of the course.

Table 1 presents simulations of optimization of equation (9) above, from which several conclusions are derived. The first illustrates the increase of productivity with practicing; subject to the restriction that practicing time can increase only until total time approaches \( T \). In addition, \( V \) increases with respect to \( t_p \), but again it can increase only until the total time approaches \( T \). The results are demonstrated by using a numerical example for the values: \( T = 100, A = 10, B = 4, \varepsilon = 2 \) and \( \alpha = 0.5 \).
Table 1  The Effect of Increasing $t_p$ on $V$

<table>
<thead>
<tr>
<th>$t_p$</th>
<th>$\hat{t}$</th>
<th>$t_L$</th>
<th>$V$</th>
<th>$n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>60</td>
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<tr>
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<td>1126.942</td>
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</tr>
</tbody>
</table>

* These values are not relevant since $\hat{t} > T$.

Based on Table 1, above, the optimal value of $V$ is achieved in the solution in which time $T$ is allocated using one single cycle, approximately at the ratio of $\frac{t_p}{t_L} = \frac{4}{1}$. Of course, these results are unique to the special values of the parameters introduced above. However, the solutions regarding the ratio between practicing time and lecturing time can be applied by running simulations of the model developed above.

4. CONCLUSIONS AND IMPLICATIONS

The relationship between lecturing and practicing in various academic professions is well known. The goal is to find a way to most effectively utilize learning time for the sake of knowledge maximization.

The distribution and number of hours required for traditional classroom teaching or for online learning must be examined so that learning is transferred and knowledge provided most effectively. In addition, the number of hours needs to be divided among hours of practice, memorization, or laboratory in order to repeat the material that students learn in the classroom. Many doubts arise concerning (i) the division of resources among the hours allocated to the various alternative factors and (ii) the length of courses.

Various academic institutions differ substantially in their teaching methods and division of time between practicing and lecturing. In order to reach an optimal time distribution, the changes that are made may sometimes not be sufficiently understood or explained. It appears that the policymakers in academic institutions may not always have an in-depth understanding of the interdependency between practice and theoretical study, and thus the decisions regarding course length or the internal course structure between lecture and practice may require further consideration.

The model seeks to take into account the interactions between lecturing and memorization, recapitulation, and practice of what has been learned in the classroom.
We try to explain the considerations in finding the desired cycle among the various time allocations in order to produce maximum student knowledge in a given amount of time. Measures need to be used to adapt the length of a course to specific parameters of lecturing, practicing and the introductions between them. These variables, in turn, also need to be allocated within a given and limited length of $T$.

One must have an in-depth understanding of the interdependency and interaction between hours of lecture and practice, in order to enable the most effective decision-making. The desired result can be achieved by utilizing available learning time in the most efficient way.

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